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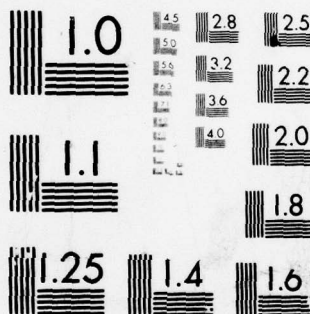


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Final Report
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June 1976

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THE UPPER-AIR ANALYSIS CAPABILITY,
FIB/UA
INTRODUCING WEIGHTED SPREADING

by

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Performed for

The Commanding Officer
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) → The Fields by Information Blending (FIB) methodology, based on fundamental and generalized information-processing concepts, is a comprehensive technique for the analysis of scalar and vector fields with a wide range of realized and potential applications. In general such applications lead to a system of linear equations which normally is too big to permit solution by explicit matrix inversion. Hitherto the sole recourse for solution of such systems has been to Successive Over-Relaxation (SOR) techniques which require a		

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Block 20. ABSTRACT (Continued)

first-guess solution and also converge at an undesirably slow rate. This Report details the development of a new technique, termed Blending by Weighted Spreading, for producing an effective solution to the system of linear equations arising from FIB applications. This technique, described in the context of its initial application in an MII-developed upper air analysis system (FIBUA), produces an effective solution to the system of linear equations which converges far more rapidly than SOR schemes alone and, of particular note, no first-guess solution is required. It is expected that all future FIB-based analysis systems, where appropriate, will utilize the new technique--Blending by Weighted Spreading.

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Appendix A

THE UPPER-AIR ANALYSIS CAPABILITY, FIB/UA INTRODUCING WEIGHTED SPREADING

1. Technological Context

1.1 Introduction

The present report covers the development of an objective analysis capability designed for the analysis of the distribution of individual, principal, atmospheric parameters in an upper-air analysis system. The system includes analysis of isobaric-height, layer-thickness, temperature, and layer-static-stability fields. Each field is a two-dimensional component field in the resolution of atmospheric mass-structure distribution variabilities.

The numerical repository for rendering the analysis of each field is a large array of grid points, regularly spaced over the regional extent of the analysis. The objective value at each grid point is representative of the objective scale of analysis resolution. The value produced at each grid point is that which is consistent with optimum accommodation of all, available, relevant, weighted, information elements, including station reports, as obtained by radiosondes, and other diverse forms of information.

This development represents a further application of the Fields by Information Blending (FIB) methodology, but with a major difference: The blending of information elements is accomplished by an innovative technique which is new to the technology of information processing. We here introduce a new analysis capability, which we designate FIB/UA for Upper-Air parameters, which incorporates blending by weighted spreading.

The broad FIB methodology is a development of Meteorology International Incorporated (MII), developed for the assimilation and blending of information for the resolution of distributions. Applications of the FIB methodology are developed in terms of minimizing disparity to information. This minimization produces a linear system of blending equations which is expressable by a symmetric, diagonally dominant, coefficient matrix.

Applications to the objective analysis of two-dimensional distributions requiring large grid-point arrays, have been developed for sea-surface temperature (FIB/SST)^[1], sea-level pressure (FIB/SLP)^[2], surface winds (FIB/UV)^[3], ocean thermal-structure parameters (FIB/OTS)^[4], and the climatology of ocean thermal-structure parameters (FIB/CLI)^[5]. The last and most recent of this series of FIB analysis programs is FIBP^[6], a version introduced in November 1974 to replace FIB/SLP in sea-level pressure analysis. All of these versions rely on SOR (Successive Over Relaxation) methods for solution of the blending system of equations.

The version identified as FIBP represents an attempt to fully exploit the technology of SOR methods insofar as could be offered by the discipline of Numerical Analysis. This effort introduced major complications in the design of FIB developments. A staggered grid-array was introduced so that the blending system of equations could be ordered to produce a block tri-diagonal matrix of submatrices. Each submatrix is penta diagonal in non-zero elements. The SOR method used in FIBP is over relaxation by blocks, with each block solved explicitly by forward elimination and back substitution. The development of FIBP also increased the number and form of information elements, for assembly and blending, without adding more non-zero matrix elements than those included in the earlier, FIB/SLP, version. The results did not come up to expectations. FIBP will surely remain the last version having reliance on SOR methods, now that the more appropriate--and far more effective--technique of blending by weighted spreading has been discovered.

At the outset of this project we planned to develop two new versions of FIB for use by the upper-air analysis system. One version was to be similar to FIBP for analysis of parameters for which measured wind profiles hold gradient estimates via geostrophic relationships. The other was to be a simpler version for parameters for which wind profiles hold negligible information. With the discovery of weighted spreading the motives for two versions no longer exist. The new version, FIB/UA, covers the intended range of parameter applications.

The discovery of weighted spreading has enabled return to simpler designs--a reversal of the trend to greater program complexity which trend has culminated in FIBP.

1.2 Discovery of Weighted Spreading

At the outset of this project to develop a new analysis capability for upper-air parameters we had several reasons for proceeding slowly. These reasons included time to study the effectiveness and suitability of SOR methods in FIB applications to large grid-point arrays.

The first suspicions of inadequate convergence in FIB applications arose in the tuning of FIBP. The reevaluation-and-gross-error-check component of FIB is sensitive to inadequacies of convergence. FIBP is more prone to convergence weakness because it is the least diagonally dominant of the class of FIB applications having 13-point-stencil equation systems. FIB/SLP and FIB/UV are also of this class, but they are generally more diagonally dominant.

The limitations of SOR capabilities were determined by experiments using FIBP. The worst of situations was presented to the SOR-blending component: A single report was introduced in the middle of the full $63 \times 63 \times 2$ grid array. The first-guess field, for initiating the SOR iterations, was 10mb different at the report location. All difference elements were defined

by the first-guess field and all weights were set within normal ranges. The solution which minimizes disparity to information is known exactly in this case: It is given by shifting the entire first-guess field by 10 mb to fit the report. Thus the first-guess field contains a uniform error of 10 mb over the whole array. The block SOR scheme accomplished about 70% of the shift at the report location, and less with increasing distance from this location. Beyond this degree of adjustment, further convergence was extremely slow.

The expansion of powers to assimilate information in a FIB application--by increasing the variety of component information elements (i.e., diagonal differences, Laplacian, etc.)--expands the number of non-zero elements in the coefficient matrix. It has been noted that this also makes the matrix less diagonally dominant, and that this slows the convergence rate of SOR techniques. The concept of slowing down the SOR-convergence rate by adding more information elements is contradictory to the basic FIB concepts of blending information. The implication is that SOR methods are not appropriate. The FIB methodology itself suggests a natural concept of spreading absolute estimates through difference estimates. And this spreading should be accelerated by additions of information elements in any and all forms. It was this conception, of how the FIB methodology should function, that we pursued to discovery of blending by weighted spreading.

Weighted spreading is characterized by the association of a weight with each successive approximation of each element of the solution. This weight is an absolute measure of the proportion of the total due influence that has arrived at that element. It is not a measure of the firmness of the solution at that point in the iterations, for further influence may subsequently arrive to substantially change the approximations. However when the ordering of elements is successive, and in alternating directions, most of the due influence is felt in only a few iterations.

Weighted spreading does not make use of any first-guess estimate to the solution for the purpose of initiating the successive approximations. The initiation stems entirely from the specified information components. Hence there is no bias introduced into the solution; characteristic components of such biases can linger in the solution even after very many iterations. But the weighted spreading solution is entirely free of any extraneous error, at any stage of the iterations. Although weighted spreading ultimately approaches the exact solution asymptotically, the initial iterations are amazingly effective in gathering information, when using successive, alternating-direction orderings.

The discovery of weighted spreading was first reported in a memorandum to the Commanding Officer of FNWC: Improved Blending Schemes for FIB Applications, dated 23 February 1976. The effect on the development of FIB/UA was reported in Quarterly Progress Report Three, dated 3 March 1976. The formulations are given in Section 3, below.

1.3 Features of FIB/UA

The numerical repository for rendering the numerical analysis is an $M \times N$ array of grid-point values. The program has been developed in terms of a regular 89×89 array, of standard hemispheric coverage on a north-polar stereographic projection.

The application of the FIB methodology includes the usual component operations: Initialization, Assembly, Blending, A^* Estimation, and Reevaluation for Recycling. FIB applications are generally recycled three times with reevaluations after the first, and after the second, cycles. The second reevaluation allows reinstatement of reports which may have been rejected in the first reevaluation on the evidence of bad reports in the ambience. However, considering the density of upper-air reports, it was

felt that three cycles were not warranted at this time. The benefits were considered to be insufficient to justify the computing of a third cycle. As now set, FIB/UA runs through two cycles, with reevaluation after the first cycle. Delivery of FIB/UA includes a Maintenance Manual and annotated program listings, to facilitate maintenance and modifications of the program.

FIB/UA includes the following component fields for the assembly of information: One field for estimates of the parameter value, six simple-difference fields for exploitation of gradient information and for exploitation of winds, and one Laplacian field for assimilation of additional information and for scale control. Each of these eight fields requires an associated weight field.

In application of blending by weighted spreading, the inclusion of the Laplacian term is costly in terms of arithmetic, and contributes information at a slower pace than do the first-difference terms. In the interests of program efficiency we have omitted the Laplacian term in the weighted spreading. But we have included all of the terms in a final blending operation in which all due-influence ratios are set to unity.

In application to analysis of layer-static-stability fields, the provision, to restrain each successive estimate in the blending operation by lower and/or upper bounds, may be exercised. By this means the information, that in the synoptic range of scale, the static stability is positive, is exploited.

FIB/UA also includes another substantial improvement over earlier versions of FIB applications--a new scheme for determining an effective approximation of the resultant firmness (i.e., A^*) of analyzed grid-point values, for use in reevaluating reports. This scheme, which has the benefits of simplicity and economy, is described in Section 3.2 below.

2. Standard FIB Formulations

2.1 The Information Fields

The material presented here, in Section 2, can be considered as standard in applications of the FIB methodology to the analysis of scalar fields using large grid-point arrays. The basic conceptions, and similar formulations, are described in the design of FIB/SLP^[2].

FIB/UA provides eight fields, each with an associated weight field, for the assembly and blending of diverse forms of information.

There is one field for the assembly of direct estimates of the scalar itself: Station report Z of associated weight A . If the report location, expressed in non-integer grid coordinates, is I, J , then the report is assembled at the ℓ, m grid point defined by:

$$\ell = I \text{ rounded} \quad , \quad m = J \text{ rounded} .$$

The report value is extrapolated from location I, J to ℓ, m according to the shape of the first-guess field. The associated assembly weight, $A_{\ell, m}$, includes a standard variance contribution for this extrapolation.

Six first-difference fields are provided for assembly of gradient estimates. Their definition, and reference locations with respect to the arbitrary ℓ, m grid point, are shown in Fig. 1. The lower-case letters denote the values of the difference estimates, and upper-case letters denote the corresponding weights.

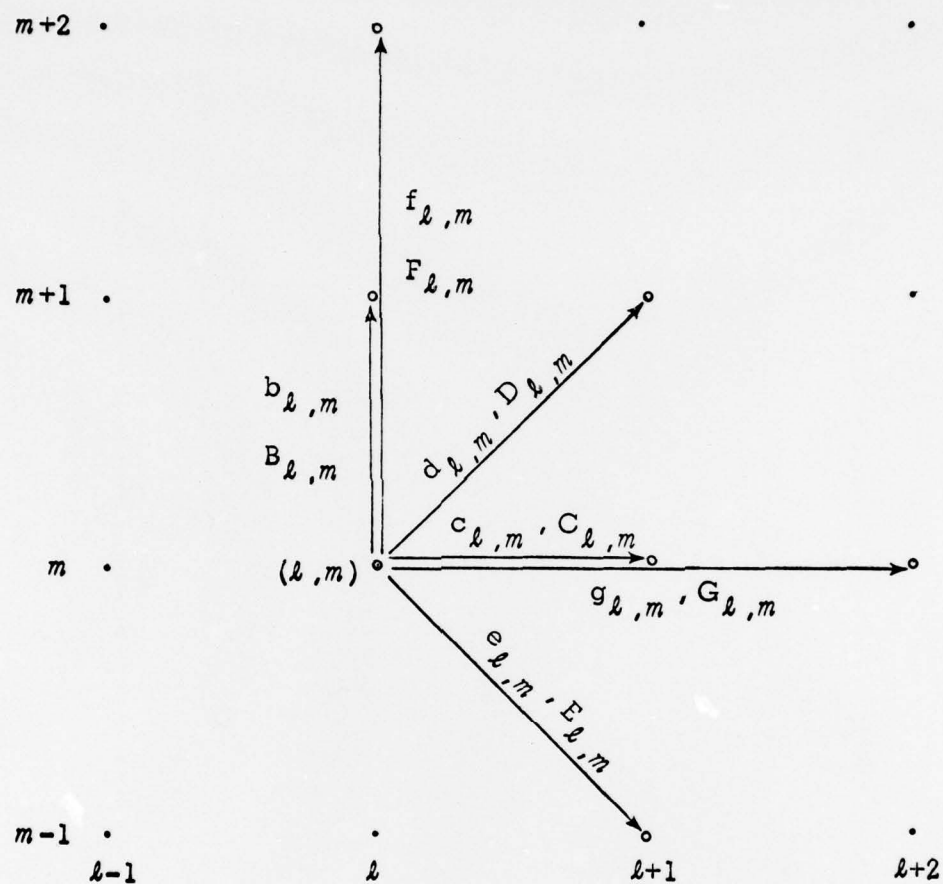


Fig. 1 Symbols and Subscripts for the Six Simple-Difference Elements

The eighth field is the Laplacian,

$$q_{l,m} \Rightarrow Z_{l,m+1} + Z_{l+1,m} + Z_{l,m-1} + Z_{l-1,m} - 4Z_{l,m}$$

of weight $Q_{l,m}$.

The Laplacian is provided for assimilation of additional information and for scale control.

The assembly weight for any element of information is a monotonic measure of the independent worth of that element of information. The weight is defined as the inverse of the associated error variance, inherent in the estimate, with respect to the object analysis value.

Estimates, at common reference locations, are combined by weighted addition. For example, an additional estimate, $Z_{l,m}^{(n)}$ of weight $A_{l,m}^{(n)}$, at the l,m grid point, is assembled as follows:

$$\text{new } A_{l,m} = A_{l,m} + A_{l,m}^{(n)} \quad (1)$$

$$\text{new } Z_{l,m} = \frac{A_{l,m} Z_{l,m} + A_{l,m}^{(n)} Z_{l,m}^{(n)}}{A_{l,m} + A_{l,m}^{(n)}} \quad (2)$$

The delivered version of FIB/UA includes provisions for several classifications of estimates, as to quality and reliability.

In application to the analysis of an isobaric height field, for example, a wind report is exploited as follows. The wind is first transformed into gradient-component estimates, expressed in terms of the two first differences,

b and c . A common weight, $B = C$, is specified accordingly. The four other first differences, and associated weights, are calculated according to the formulas:

$$d = b+c \quad , \quad e = -b+c \quad , \quad f = 2b \quad , \quad g = 2c \quad (3)$$

$$D = \frac{1}{2}B \quad , \quad E = \frac{1}{2}C \quad , \quad F = \frac{1}{4}B \quad , \quad G = \frac{1}{4}C \quad . \quad (4)$$

The first-difference estimates are each assembled at the nearest reference locations appropriate to the site of the observation. If the report location is I,J then the respective reference locations are obtained as follows:

For b	$l = I$ rounded	$m = J$ truncated
c	$l = I$ truncated	$m = J$ rounded
d	$l = I$ truncated	$m = J$ truncated
e	$l = I$ truncated	$m = J$ truncated +1
f	$l = I$ rounded	$m = J$ rounded -1
g	$l = I$ rounded -1	$m = J$ rounded

The resulting set of six locations differs according to which quarter area module the report falls into. This gives considerable resolution for the exploitation of wind reports.

The assembly of gradient estimates derived from winds, and wind profiles, raises a special consideration. Implicit in every transformation of a wind profile into a gradient estimate is the error variance that is inherent in the relationship on which the transformation is based. If in each difference field there is rarely more than one such estimate assembled

at any one grid point then this variance contribution can be included in the assembly of each estimate. But, if wind reports are more dense--if 2 or more winds are assembled at any 1 grid point--then the gradient estimates are first weighted only for wind errors. The error variance inherent in the wind-to-gradient transformation must be added after the assembly of the gradient estimates in order to limit the assembled weight. The assembled weight fields are then reduced by addition of the relationship variance at each and every grid point. This treatment recognizes the spatial covariance of the relationship variance.

The delivered version of FIB/UA derives first-guess fields, for each of the eight contributing information fields, from a single specified first-guess field for the object parameter. The corresponding, uniform weight fields are specified by the program according to the object parameter and the design of the upper-air analysis system. These fields are then ready for assembly of reports. However, gradient estimates derived from winds should first be separately assembled with wind variances only, and then, with the added variance for the relationship, they may be combined with the first-guess fields. There is no provision for the assembly of estimates into the Laplacian field.

2.2 The Error Functional

The desired solution, which we denote for the arbitrary l, m grid point by $Z_{l, m}^*$, is that solution which gives a best fit to the ensemble of eight weighted information fields. We have established in earlier work that the best fit is given by the solution which minimizes the error functional \textcircled{E} , as defined by Eq. (5). This solution is consistent with the fundamental rules on which the FIB methodology is based.

$$\begin{aligned}
\textcircled{E} \equiv & \sum_{l,m} \left\{ A_{l,m} (Z_{l,m}^* - Z_{l,m})^2 \right. \\
& + B_{l,m} (Z_{l,m+1}^* - Z_{l,m}^* - b_{l,m})^2 \\
& + C_{l,m} (Z_{l+1,m}^* - Z_{l,m}^* - c_{l,m})^2 \\
& + D_{l,m} (Z_{l+1,m+1}^* - Z_{l,m}^* - d_{l,m})^2 \\
& + E_{l,m} (Z_{l+1,m-1}^* - Z_{l,m}^* - e_{l,m})^2 \\
& + F_{l,m} (Z_{l,m+2}^* - Z_{l,m}^* - f_{l,m})^2 \\
& + G_{l,m} (Z_{l+2,m}^* - Z_{l,m}^* - g_{l,m})^2 \\
& \left. + Q_{l,m} (Z_{l,m+1}^* + Z_{l+1,m}^* + Z_{l,m-1}^* + Z_{l-1,m}^* - 4Z_{l,m}^* - q_{l,m})^2 \right\} \quad (5)
\end{aligned}$$

2.3 The System of Blending Equations

The minimum value of the error functional occurs when

$$\frac{\partial \textcircled{E}}{\partial Z_{\ell, m}^*} = 0 \quad (6)$$

simultaneously, for every element $Z_{\ell, m}^*$ of the solution field. Equation (6) yields one equation per grid point--producing a simultaneous system of linear equations--the system of blending equations. The blending system may be expressed by the general form of each member equation. This general form results directly from the development of Eq. (6). The result is Eq. (7)

The Blending Equation:

$$\begin{aligned}
& S_{l,m} \cdot Z_{l,m}^* = A_{l,m} \cdot Z_{l,m} \\
& + B_{l,m} \cdot (Z_{l,m+1}^* - b_{l,m}) + B_{l,m-1} \cdot (Z_{l,m-1}^* + b_{l,m-1}) \\
& + C_{l,m} \cdot (Z_{l+1,m}^* - c_{l,m}) + C_{l-1,m} \cdot (Z_{l-1,m}^* + c_{l-1,m}) \\
& + D_{l,m} \cdot (Z_{l+1,m+1}^* - d_{l,m}) + D_{l-1,m-1} \cdot (Z_{l-1,m-1}^* + d_{l-1,m-1}) \\
& + E_{l,m} \cdot (Z_{l+1,m-1}^* - e_{l,m}) + E_{l-1,m+1} \cdot (Z_{l-1,m+1}^* + e_{l-1,m+1}) \\
& + F_{l,m} \cdot (Z_{l,m+2}^* - f_{l,m}) + F_{l,m-2} \cdot (Z_{l,m-2}^* + f_{l,m-2}) \\
& + G_{l,m} \cdot (Z_{l+2,m}^* - g_{l,m}) + G_{l-2,m} \cdot (Z_{l-2,m}^* + g_{l-2,m}) \\
& + 16 Q_{l,m} \cdot \frac{1}{4} (Z_{l,m+1}^* + Z_{l+1,m}^* + Z_{l,m-1}^* + Z_{l-1,m}^* - q_{l,m}) \\
& + Q_{l-1,m} \cdot (4Z_{l-1,m}^* - Z_{l-1,m+1}^* - Z_{l-1,m-1}^* - Z_{l-2,m}^* + q_{l-1,m}) \\
& + Q_{l,m-1} \cdot (4Z_{l,m-1}^* - Z_{l+1,m-1}^* - Z_{l,m-2}^* - Z_{l-1,m-1}^* + q_{l,m-1}) \\
& + Q_{l+1,m} \cdot (4Z_{l+1,m}^* - Z_{l+1,m+1}^* - Z_{l+2,m}^* - Z_{l+1,m-1}^* + q_{l+1,m}) \\
& + Q_{l,m+1} \cdot (4Z_{l,m+1}^* - Z_{l,m+2}^* - Z_{l+1,m+1}^* - Z_{l-1,m+1}^* + q_{l,m+1}) \quad (7)
\end{aligned}$$

In Eq. (7), $S_{l,m}$ denotes the sum of all weights which appear as coefficient--the coefficients precede each dot--of each term on the right-hand side:

$$S_{l,m} \equiv A_{l,m} + B_{l,m} + B_{l,m-1} + \dots Q_{l,m+1} \quad (8)$$

It is convenient to use this notation of dots to separate coefficients and estimates. In effect, Eq. (7) may also represent Eq. (8). This notation for expressing a pair of equations will come up again later, in the context of blending by weighted spreading.

The system of blending equations may be expressed in matrix notation:

$$\underset{\sim}{M} \underset{\sim}{Z}^* = \underset{\sim}{R} \quad (9)$$

Equation (7) represents an arbitrary row of Eq. (9). The formal solution is

$$\underset{\sim}{Z}^* = \underset{\sim}{M}^{-1} \underset{\sim}{R}$$

but the matrix is generally far too large for routine explicit inversion.

Equation (9) is the beginning of formalism for the development of SOR schemes of approximate solution. We have traveled this route before, several times, in complexities ranging from FIB/SLP (successive point-by-point relaxation in a 5 subset ordering) to FIBP (Block relaxation in alternating order). But this time our main interest lies in the introduction of blending by weighted spreading. The formulations are given in Section 3.

2.4 Boundary Treatment

The addition of a full perimeter, two grid points deep, to each field, takes care of all complications at the lateral boundaries of the grid. This dummy exterior, in effect, expands the L by M grid to L+4 by M+4. The dummy points have the indexes:

$$\begin{aligned} \ell &= 1, 2, L+3, L+4 && \text{for all } m, \text{ and} \\ m &= 1, 2, M+3, M+4 && \text{for } \ell = 3 \text{ through } L+2. \end{aligned}$$

All field values are set to zero at the dummy exterior points. In addition, the weights of those information elements which connect interior elements to exterior elements are also set to zero, permanently:

$$\begin{aligned} B_{\ell,m} &= 0 && \text{for } m = M+2, \text{ and } \ell = 3 \text{ through } L+2. \\ C_{\ell,m} &= 0 && \text{for } \ell = L+2, \text{ and } m = 3 \text{ through } M+2. \\ D_{\ell,m} &= 0 && \text{for } \ell = L+2, \text{ and } m = 3 \text{ through } M+2, \text{ and} \\ &&& \text{for } m = M+2, \text{ and } \ell = 3 \text{ through } L+1. \\ E_{\ell,m} &= 0 && \text{for } \ell = L+2, \text{ and } m = 3 \text{ through } M+2, \text{ and} \\ &&& \text{for } m = 3, \text{ and } \ell = 3 \text{ through } L+1. \\ F_{\ell,m} &= 0 && \text{for } m = M+1, M+2, \text{ and } \ell = 3 \text{ through } L+2. \\ G_{\ell,m} &= 0 && \text{for } \ell = L+1, L+2, \text{ and } m = 3 \text{ through } M+2. \\ Q_{\ell,m} &= 0 && \text{for } \ell = 3, L+2, \text{ and } m = 3 \text{ through } M+2, \text{ and} \\ &&& \text{for } m = 3, M+2, \text{ and } \ell = 4 \text{ through } L+1. \end{aligned}$$

With these provisions, all computational loops in the program can range over the interior points without change near, and at, the boundaries.

In the actual program these provisions are taken care of by simply setting the weight to zero of all components which extend outside the grid or which are undefined. In this way no dummy points need to be added.

2.5 Theoretical A^*

The concept of a resultant reliability weight, $A_{\ell,m}^*$, which is a measure of the firmness of the solution element, $Z_{\ell,m}^*$, is basic to the FIB methodology. Formally, the A^* elements appear, inverted, as the diagonal elements of the inverse matrix in Eq. (10). Concepts, and methods of estimating A^* , have been developed in earlier reports^{[2],[6]}. These earlier methods have the same faults as SOR schemes; they are costly in arithmetic and are not always effective. In connection with blending by weighted spreading, we have developed a simpler, more effective, and more controllable scheme for approximating A^* .

A good approximation to A^* not only rounds out the utility of the analysis solution. It is also used within the program itself, in the reevaluation component.

2.6 Reevaluation of Reports

The reevaluation of reports follows preliminary solutions of the fields of Z^* , and corresponding weights, A^* . The ingredients which enter into the reevaluation of a report include the following items:

Z_n	the assembly value is the report value extrapolated to the nearest grid point by the shape of the first-guess field
A_n	the assembly weight by which Z_n is assembled at the nearest grid point
Z^*	the resultant analysis value at this grid point
A^*	the resultant analysis weight at this grid point
A_c	the purported weight for the particular class of reports

The independent information against which the report may be compared is the analysis resultant less the contribution made by the report. This is called the background information:

$$A_B = A^* - A_n \quad (11)$$

$$Z_B = \frac{A^* Z^* - A_n Z_n}{A^* - A_n} \quad (12)$$

The difference between Z_n and Z_B may be expressed in units, λ , of expected difference:

$$\lambda_n^2 = \frac{A_c A_B}{A_c + A_B} \left(Z_n - Z_B \right)^2 \quad (13)$$

Reevaluation consists of replacing the assembly weight, A_n , by a reevaluated assembly weight, A_{nR} :

$$A_{nR} = \left(\frac{k - \lambda_n^2}{k} \right) A_n \geq 0 \quad (14)$$

where $k \equiv \overline{\lambda^2}$, an upper bound for acceptance.

There is really no strict basis for reevaluating winds. We can, however, reduce or reject the weights of winds which are not representative of the geostrophic wind.

Let b_n and c_n represent the two locally-orientated first differences derived from a wind, or wind profile, at report location I, J , and let B_n be the common weight. The resultant geostrophic equivalents of the analysis are obtained by interpolation in the field of Z^* :

$$b^* = Z_{J+1/2}^* - Z_{J-1/2}^* \quad (15)$$

$$c^* = Z_{I+1/2}^* - Z_{I-1/2}^* \quad (16)$$

The reevaluation is defined by

$$B_{nR} = (1 - \lambda^2) B_n \cong 0 \quad (17)$$

where

$$\lambda^2 \equiv B_n \left[(b^* - b_n)^2 + (c^* - c_n)^2 \right] \quad (18)$$

3. New FIB Formulations

3.1 Blending by Weighted Spreading

Blending by Weighted Spreading is a new technique for producing an effective solution to the system of blending equations, with economy and efficiency in computations. The concept is that of a process of successive approximations in which absolute resolution, which stems from all grid points where $A_{l,m} \neq 0$, spreads through spatial-difference estimates to blend throughout the region. The concept is based on understanding the system of blending equations. Equation (7) states that the solution, $Z_{l,m}^*$, is the resultant of 18 weighted estimates, one of which is forcing and the rest are implicit estimates drawn from the ambient estimates.

In common applications of the method of successive approximations, the estimates are combined as weighted, without differentiation as to the accuracy of each separate estimate in its progress toward solution. Blending by weighted spreading is achieved by introduction of a progress factor for each successive estimate. A low progress factor reduces spreading. A high progress factor increases spreading.

We now have a better appreciation as to how the addition of more finite-difference information elements retards solution by SOR schemes. The initiation of SOR schemes generally begins with a first guess to the solution. This first-guess field is regionally biased to the extent that the error is spatially correlated. Without progress factors these biased ambient estimates overwhelm any forcing estimates, and retard the progress toward solution. Blending by weighted spreading does not involve any extraneous first guess. The process begins with the forcing elements.

As a matter of convenience, and for other reasons to be explained later, we here introduce the development of weighted spreading without the Laplacian information elements. That is, for the time being, consider $Q \equiv 0$ for all ℓ, m . This reduces Eq. (7) to a combination of one forcing term and the contributions of 12 other estimates drawn from ambient grid-point estimates via first-difference estimates. Each estimate via a first difference is based on one ambient grid-point estimate.

$$\begin{aligned}
& \alpha_{l,m}^{(R+1)} \cdot Z_{l,m}^{(R+1)} = A_{l,m} \cdot Z_{l,m} \\
& + B_{l,m} \frac{\alpha_{l,m+1}^{(R)}}{S_{l,m+1}} \cdot \left(Z_{l,m+1}^{(R)} - b_{l,m} \right) + B_{l,m-1} \frac{\alpha_{l,m-1}^{(R)}}{S_{l,m-1}} \cdot \left(Z_{l,m-1}^{(R)} + b_{l,m-1} \right) \\
& + C_{l,m} \frac{\alpha_{l+1,m}^{(R)}}{S_{l+1,m}} \cdot \left(Z_{l+1,m}^{(R)} - c_{l,m} \right) + C_{l-1,m} \frac{\alpha_{l-1,m}^{(R)}}{S_{l-1,m}} \cdot \left(Z_{l-1,m}^{(R)} + c_{l-1,m} \right) \\
& + D_{l,m} \frac{\alpha_{l+1,m+1}^{(R)}}{S_{l+1,m+1}} \cdot \left(Z_{l+1,m+1}^{(R)} - d_{l,m} \right) + D_{l-1,m-1} \frac{\alpha_{l-1,m-1}^{(R)}}{S_{l-1,m-1}} \cdot \left(Z_{l-1,m-1}^{(R)} + d_{l-1,m-1} \right) \\
& + E_{l,m} \frac{\alpha_{l+1,m-1}^{(R)}}{S_{l+1,m-1}} \cdot \left(Z_{l+1,m-1}^{(R)} - e_{l,m} \right) + E_{l-1,m+1} \frac{\alpha_{l-1,m+1}^{(R)}}{S_{l-1,m+1}} \cdot \left(Z_{l-1,m+1}^{(R)} + e_{l-1,m+1} \right) \\
& + F_{l,m} \frac{\alpha_{l,m+2}^{(R)}}{S_{l,m+2}} \cdot \left(Z_{l,m+2}^{(R)} - f_{l,m} \right) + F_{l,m-2} \frac{\alpha_{l,m-2}^{(R)}}{S_{l,m-2}} \cdot \left(Z_{l,m-2}^{(R)} + f_{l,m-2} \right) \\
& + G_{l,m} \frac{\alpha_{l+2,m}^{(R)}}{S_{l+2,m}} \cdot \left(Z_{l+2,m}^{(R)} - g_{l,m} \right) + G_{l-2,m} \frac{\alpha_{l-2,m}^{(R)}}{S_{l-2,m}} \cdot \left(Z_{l-2,m}^{(R)} + g_{l-2,m} \right)
\end{aligned}$$

(19)

The process of Blending by Weighted Spreading is defined by Eq. (19). This equation is written in our convention of dots between the coefficient weights and the estimates. According to this convention Eq. (19) defines a second equation for the coefficients:

$$\alpha_{\ell,m}^{(R+1)} = A_{\ell,m} + B_{\ell,m} \frac{\alpha_{\ell,m+1}^{(R)}}{S_{\ell,m+1}} + B_{\ell,m-1} \frac{\alpha_{\ell,m-1}^{(R)}}{S_{\ell,m-1}} + \dots + G_{\ell-2,m} \frac{\alpha_{\ell-2,m}^{(R)}}{S_{\ell-2,m}} \quad (20)$$

The superscript R in parentheses is used to denote the R-th estimate in a sequence of successive approximations. The process consists of proceeding from grid point to grid point, in any preferred ordering, computing new estimates $\alpha_{\ell,m}^{(R+1)}$ and $Z_{\ell,m}^{(R+1)}$, by explicit solution of Eqs. (20) and (19) in that order. Each pass at all the grid points advances R by 1.

The successive approximations are initiated by setting

$$\begin{aligned} \alpha_{\ell,m}^{(0)} &\equiv 0 \\ Z_{\ell,m}^{(0)} &\equiv 0 \end{aligned} \quad (21)$$

No extraneous first-guess field is introduced.

The progress factor which is carried along with each successive approximation, $Z_{\ell,m}^{(R)}$, may be denoted by

$$\beta_{\ell,m}^{(R)} \equiv \alpha_{\ell,m}^{(R)} / S_{\ell,m} \quad (22)$$

Equation (7) states that the total weight of all influences due to arrive at the arbitrary l, m grid point is given by $S_{l, m}$. At any stage, $\alpha_{l, m}^{(R)}$ represents the portion of this due influence that has arrived. The progress factor, $\beta_{l, m}^{(R)}$, begins at zero and grows monotonically with each pass, to asymptotically approach 1.

Any ordering of grid points is permitted. Our preferred ordering is successive in l , increasing or decreasing, within a successive ordering of m , increasing or decreasing. A complete set of 4 such passes consists of:

- Pass 1: l increasing , m increasing
- Pass 2: l decreasing , m decreasing
- Pass 3: l decreasing , m increasing
- Pass 4: l increasing , m decreasing

The progress factor, $\beta_{l, m}^{(R)}$, is an absolute measure of the proportion of the total due influence that has arrived at the l, m location. It is not a measure of the firmness of the value, $Z_{l, m}^{(R)}$, however. Further due influence may subsequently arrive to produce a pronounced change in the value. Pronounced subsequent changes are possible when the iterations are performed by simultaneous rather than successive displacement, allowing the propagation to spread by only one or two grid lengths per pass. Simultaneous displacement would allow distant influences to arrive as a wave of change in a region sparse in local influences.

Pronounced changes are very unlikely to occur after only a few multi-directional successive iterations have been effected. A set of 4 such passes spreads all influences to all locations--although not immediately to their full measure. Subsequent changes will generally be very gradual and very minor, as β asymptotically approaches unity everywhere.

Constraints on the solution, such as the placing of a lower bound in the analysis of layer static-stability fields, can be applied in the course of the blending, by forcing each successive approximation to stay within the imposed bound(s).

The Laplacian element in the error functional, Eq. (5), gives rise to five terms in the blending equation, Eq. (7). These five compound estimates can be included in the formulations of weighted spreading. Inclusion requires the imposition of a suitable progress factor in the coefficient of each estimate. We consider the smallest $\beta^{(R)}$, among those for the four values which enter into each estimate, to be an effective factor.

We have omitted the Laplacian terms, in the blending by weighted spreading, for several reasons:

1. Inclusion of the five Laplacian terms almost doubles the computational work per pass.
2. The Laplacian terms contribute absolute resolution at a slower pace than do the first-difference terms.
3. The Laplacian information can be included in a final supplemental blending in which all progress factors are set to unity.
4. Exclusion of the Laplacian terms, has enabled formulation and use of an effective approximation to the resultant reliability weight field, $A_{l,m}^*$, in just one computational pass.

The final blending operation, to which item 3 above refers, resembles an earlier SOR scheme--that used in FIB/SLP^[2]. Its use here, however, is only for adding finishing touches to the blending solution obtained by weighted spreading, as an economical way of adding the Laplacian information. No regional biases remain; none are introduced by extraneous

first guesses. The Laplacian information is introduced gradually. Each full pass at the array is made in terms of five subsets to produce an accelerated form of simultaneous displacement. The relaxation factor, ω , is set less than 1 in the initial passes, rising in the sequence $\omega = 0.3, 0.6, 1.0, 1.2$, and continuing at 1.5 with the fifth pass.

3.2 Approximate A^*

The combined weight, $S_{l,m}$, of the total influence due at grid point l,m does not all represent independent information. We can only be certain about the independence of the $A_{l,m}$ contribution. The additional weight, $S_{l,m} - A_{l,m}$, represents, at least to some degree, a feedback of the $A_{l,m}$ contribution. We can conclude that

$$A_{l,m} \leq A_{l,m}^* < S_{l,m} \quad . \quad (23)$$

In developing a simplified approximation to A^* , it is important to keep two considerations in mind:

- (1) The approximation should be representative of the gathering of independent information that has actually been gathered by the weighted spreading, in the specified total of R passes, giving the solution

$$Z_{l,m}^* \equiv Z_{l,m}^{(R)} \quad . \quad (24)$$

- (2) An overestimation of A^* is preferred for use in the reevaluation of reports. Underestimation can result in rejection of good reports.

The first of these two considerations permits a further narrowing of the limits expressed by Eq. (23). According to the second consideration

this narrowing is valuable because it lowers the upper bound on $A_{\ell,m}^*$:

$$A_{\ell,m} \leq A_{\ell,m}^* < \alpha_{\ell,m}^{(R)} < S_{\ell,m} . \quad (25)$$

The upper bound on $A_{\ell,m}^*$ can be brought down even lower by removal of the direct feedback of information through the first-difference estimates. This reduced upper bound provides a simple, effective approximation to $A_{\ell,m}^*$. The formula is given by Eq. (26).

$A_{l,m}^*$ \cong $A_{l,m}$

$$\begin{aligned} & + \frac{B_{l,m}}{S_{l,m+1}} \left(\alpha_{l,m+1}^{(R)} - \frac{B_{l,m}}{S_{l,m}} \alpha_{l,m}^{(R-1)} \right) + \frac{B_{l,m-1}}{S_{l,m-1}} \left(\alpha_{l,m-1}^{(R)} - \frac{B_{l,m-1}}{S_{l,m}} \alpha_{l,m}^{(R-1)} \right) \\ & + \frac{C_{l,m}}{S_{l+1,m}} \left(\alpha_{l+1,m}^{(R)} - \frac{C_{l,m}}{S_{l,m}} \alpha_{l,m}^{(R-1)} \right) + \frac{C_{l-1,m}}{S_{l-1,m}} \left(\alpha_{l-1,m}^{(R)} - \frac{C_{l-1,m}}{S_{l,m}} \alpha_{l,m}^{(R-1)} \right) \\ & + \frac{D_{l,m}}{S_{l+1,m+1}} \left(\alpha_{l+1,m+1}^{(R)} - \frac{D_{l,m}}{S_{l,m}} \alpha_{l,m}^{(R-1)} \right) + \frac{D_{l-1,m-1}}{S_{l-1,m-1}} \left(\alpha_{l-1,m-1}^{(R)} - \frac{D_{l-1,m-1}}{S_{l,m}} \alpha_{l,m}^{(R-1)} \right) \\ & + \frac{E_{l,m}}{S_{l+1,m-1}} \left(\alpha_{l+1,m-1}^{(R)} - \frac{E_{l,m}}{S_{l,m}} \alpha_{l,m}^{(R-1)} \right) + \frac{E_{l-1,m+1}}{S_{l-1,m+1}} \left(\alpha_{l-1,m+1}^{(R)} - \frac{E_{l-1,m+1}}{S_{l,m}} \alpha_{l,m}^{(R-1)} \right) \\ & + \frac{F_{l,m}}{S_{l,m+2}} \left(\alpha_{l,m+2}^{(R)} - \frac{F_{l,m}}{S_{l,m}} \alpha_{l,m}^{(R-1)} \right) + \frac{F_{l,m-2}}{S_{l,m-2}} \left(\alpha_{l,m-2}^{(R)} - \frac{F_{l,m-2}}{S_{l,m}} \alpha_{l,m}^{(R-1)} \right) \\ & + \frac{G_{l,m}}{S_{l+2,m}} \left(\alpha_{l+2,m}^{(R)} - \frac{G_{l,m}}{S_{l,m}} \alpha_{l,m}^{(R-1)} \right) + \frac{G_{l-2,m}}{S_{l-2,m}} \left(\alpha_{l-2,m}^{(R)} - \frac{G_{l-2,m}}{S_{l,m}} \alpha_{l,m}^{(R-1)} \right) \end{aligned}$$

(26)

The application of Eq. (26) requires saving the fields of $\alpha^{(R-1)}$ and $\alpha^{(R)}$ from the blending by weighted spreading. As an expedient the delivered program saves only $\alpha^{(R)}$ which it also uses in place of $\alpha^{(R-1)}$. If the number of passes, R , is adequate, there should be little difference.

4. Design of the Program

A simple block diagram, Fig. 2, serves to illustrate the component operations of the program. The program has been designed in terms of a driving subroutine FIBUA, which calls the component operations as subroutines. Program details are provided by the Program Maintenance Manual, and the program listing which is annotated by program comments.

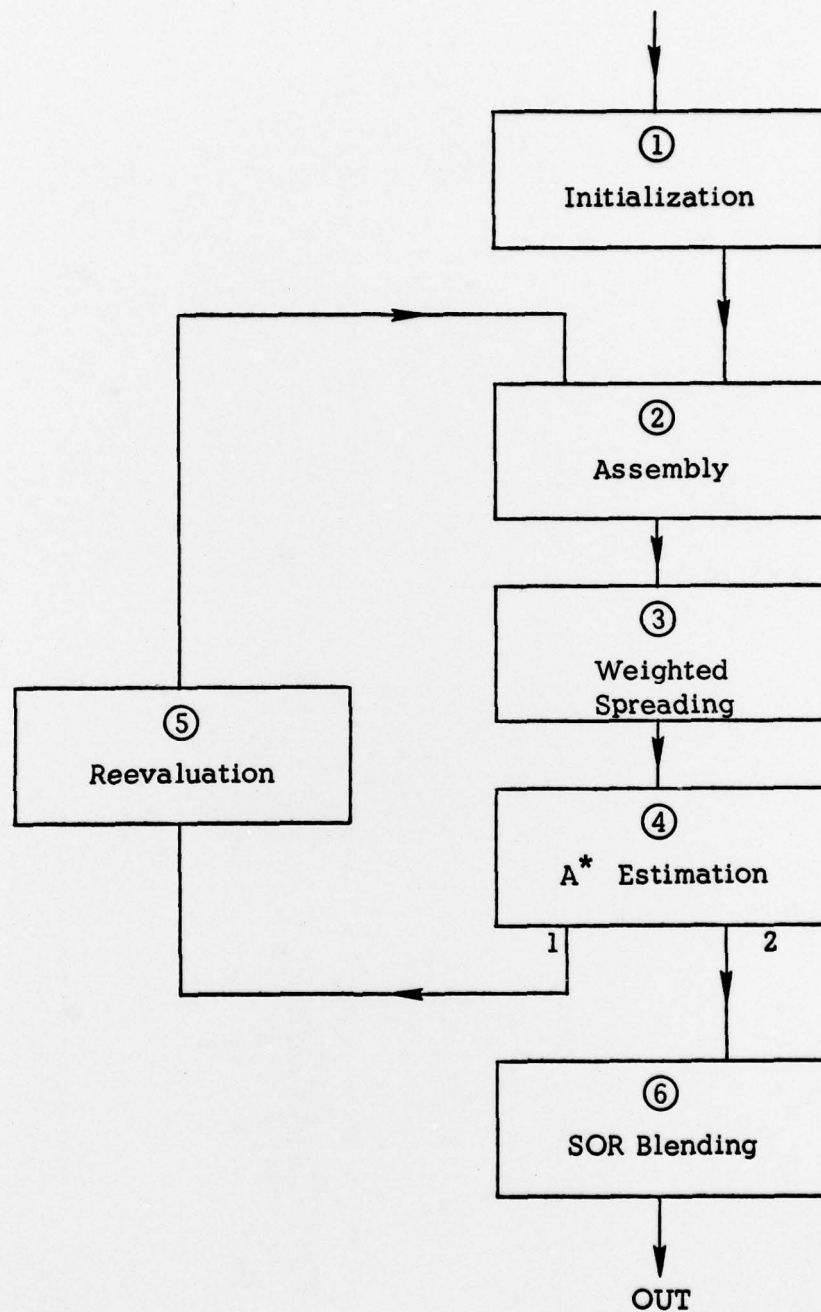


Fig. 2 Schematic Block Diagram of Component Operations

5. References

- [1] Holl, Manfred M., Bruce R. Mendenhall and Charles E. Tilden, 1971; "Technical Developments for Operational Sea Surface Temperature Analysis with Capability for Satellite Data Input", Final Report, (Contract No. N62306-70-C-0334, Naval Weapons Engineering Support Activity, Detachment FAMOS), Meteorology International Incorporated, Monterey, California, 73 pp.
- [2] Holl, Manfred M. and Bruce R. Mendenhall, 1971; "Fields by Information Blending, Sea-Level Pressure Version", Final Report, (Contract No. N66314-70-C-5226, Fleet Numerical Weather Central), Meteorology International Incorporated, Monterey, California, 66 pp.
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- [3] Weigle, William F., Bruce R. Mendenhall and Manfred M. Holl, 1973; "Development of FIB/UV-MED, A Capability for the Objective Analysis of Surface Wind with adaptation to the Greater Mediterranean Region", Technical Summary (Contract No. N66314-73-C-1444, Environmental Prediction Research Facility), Meteorology International Incorporated, Monterey, California, 39 pp.
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- [5] Weigle, William F. and Bruce R. Mendenhall, 1974; "Climatology of the Upper Thermal-Structure of the Seas", Technical Summary, (Contract No. N66314-73-C-1698, Fleet Numerical Weather Central), Meteorology International Incorporated, Monterey, California, 79 pp.
- [6] Weigle, William F. and Bruce R. Mendenhall, 1974; "The Sea-Level Pressure Analysis Capability, FIB/SLP: Improvements, Refinements, and Generalizations", Technical Summary, (Contract No. N66314-74-C-1528, Fleet Numerical Weather Central), Meteorology International Incorporated, Monterey, California, 37 pp.

APPENDIX A

Documentation Material Suitable
for Inclusion in the
Naval Weather Service
Numerical Environmental Products Manual

John G. Locke

June 1976

THE FIB UPPER-AIR ANALYSIS (FIB/UA)

A. Development of the FIB Methodology

The methodology of Fields by Information Blending (FIB) was developed for the blending and assimilation of information to resolve variabilities in the distribution of an object parameter. The various sources and levels of information are treated through a realistic assessment of the potential contribution of each datum to the total information pool. Development of the FIB techniques from applications to analysis of sea-surface temperature, through sea-level pressure, surface winds, ocean thermal structure and ocean parameters, satellite clear-column radiances, and further refinements applied to sea-level pressure, and the discovery of Blending by Weighted Spreading used in the upper-air analysis, provides a history of the refinements of the advancing methodology of FIB. All versions of FIB applications are based on the realization of information as a metered quantity. The weighting values associated with the flow and blending of information elements are an absolute measure of the value of the elements in each step of the blending process.

Previous versions of the FIB technique (prior to FIB/UA) all rely on Successive Over Relaxation (SOR) methods for solution of the blending system of equations. Version FIBP is a complex system representing full exploitation of the Numerical Analysis technology of SOR. FIB/UA introduces an advance beyond SOR capabilities--much more appropriate to the FIB methodology.

The basic method of FIB analysis (See Section 3.10) remains relevant. The upper-air analysis, FIB/UA, however, incorporating a refinement over other versions is at the forefront of information processing methodology. The new technique, Blending by Weighted Spreading eclipses the previous SOR method in assuring rapid, effective convergence under all conditions.

B. Program Input

Subroutine FIBUA is the controlling program for the FIB/UA system. The input requirements (in the FORTRAN argument order) are:

1. A first-guess field for the object parameter (the parameter being analyzed). The 20-word identification block is used as described in the FIBUA Program Maintenance Manual.
2. An array containing the object data list.
3. An array containing data relating to the object parameter first difference (the wind information), if any.
4. An array containing relative weights and subroutine parameter settings.

C. Program Computation

Basic program steps listed here are described in some detail in Section 3.10.

Computational steps:

1. Initialization
2. Assembly
3. Blending by Weighted Spreading
4. Estimation of the reliability weight field
5. Reevaluation
6. Repetition of steps 2, 3, and 4
7. SOR Blending

D. Program Limitations and Verification

To assist verification efforts and FIB/UA system tuning, a summary is printed which indicates the total number of reports, the number of rejected reports, and the mean reported and analyzed values. Other parameters indicating degree of data accommodation are also printed. Possible bias in the analysis should be quickly indicated by these statistics. Additionally, details of the reevaluation are stored in the data list for optional printout.

E. Program Output

The analyzed field, and the data lists with reject bits are returned in the same locations as the corresponding inputs. The adjunct reliability fields, used internally, are not saved or output, but they are important in the development and application of advanced techniques for exploitation of satellite sensing capabilities and other sources of relevant information.

F. References

- [1] Holl, Manfred M., 1976; "The Upper-Air Analysis Capability, FIB/UA Introducing Weighted Spreading", Final Report (Contract No. N00228-75-C-2374, Fleet Numerical Weather Central, Meteorology International Incorporated, Monterey, California, 32 pp. plus Appendix A.
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